Study Guide for the MQE

**Topic I - RATIONAL EXPRESSIONS**

Addition, subtraction, multiplication, and division of rational expressions; complex rational expressions.

**Topic II - EXPONENTS AND RADICALS**

Laws of exponents; calculations involving exponents and radicals; conversion between radicals and exponents; rationalization techniques.

**Topic III - LINEAR EQUATIONS, INEQUALITIES AND ABSOLUTE VALUE**

Solution of one and two linear equations; solution of equations reducible to linear equations; graphs of linear equations; solution of linear inequalities; simplification of expressions involving absolute value; solution of equations and inequalities involving absolute value.

**Topic IV - POLYNOMIALS AND POLYNOMIAL EQUATIONS**

Polynomial algebra; factorization; completing the square; solution of polynomial equations by factoring; solution of quadratic equations by quadratic formula; graphs of quadratic equations; solution of quadratic inequalities.

**Topic V - FUNCTIONS**

Function concept and notation; function evaluation and composition; graphs of translations, reflections, and absolute value of functions.

**Topic VI - TRIGONOMETRY**

Right angle trigonometry; trigonometric functions as circular functions; radian and degree measure; special angles; trigonometric identities and equations; graphs of trigonometric functions.

**Topic VII - LOGARITHMIC AND EXPONENTIAL FUNCTIONS**

Definition and laws of logarithms; evaluation of logarithmic expressions; graphs of logarithmic and exponential functions; inverse relation between logarithm and exponential function; logarithmic and exponential equations.

**Topic VIII - WORD PROBLEMS**

Problems involving percent, average, and ratio and proportion; problems leading to linear and quadratic equations; problems from geometry.
Sample Test

Topic I - RATIONAL EXPRESSIONS

1. Divide and simplify
\[
(x - \frac{1}{x}) \div (1 - \frac{1}{x})
\]

2. Perform the indicated operations and simplify
\[
\frac{2}{x} - \frac{1}{x^2 - 4}
\]

3. Simplify
\[
\frac{5u^2 - 4v}{\frac{v^3}{uv}} (3v^2)
\]

4. Subtract and simplify
\[
\frac{10}{x^2 - 4} - \frac{3 - x}{x^2 + 2x}
\]

5. Add and simplify
\[
\frac{1}{(a - b)(a + 2b)} + \frac{1}{(a + 2b)(a - 3b)} + \frac{1}{(a - 3b)(a - b)}
\]

6. Perform the indicated operations and simplify
\[
(x - \frac{1}{y})^{-1}
\]

7. Divide and simplify
\[
\frac{a^2b}{a - b} \div \frac{a + b}{a^2 - b^2}
\]

8. Perform the indicated operations and simplify
\[
\frac{x}{x - 3} - \frac{2x}{x^2 - 2x - 3}
\]
\[
\frac{2}{x + 1} - \frac{1}{x}
\]
Topic II - Exponents and Radicals

1. Evaluate $32^{-3/5}$
2. Simplify $\sqrt{10}/15$
3. Simplify $\sqrt{(u^2 - v^2)(u + v)}$, $u > v > 0$
4. Simplify $\sqrt{16x^2 + 36y^2}$
5. Simplify $\frac{(5ab^2)(2a^3b)^2}{a^3b^2}$
6. Simplify $\frac{x^{-2}y^{-2}}{x^{-1}y^{-2}z^0}$
7. Simplify $\frac{u^2 - 3}{x^{u - 3}}$
8. Simplify $(x^3 - 1)^{a + 1}$
9. Rationalize $\frac{a}{\sqrt[3]{4}}$
10. Simplify $\frac{9x^{3/2}y^{5/2}}{81x^{-1/2}y^{-1}}$
11. Simplify $(-27x^{12}y^{-18})^{-2/3}$
12. Simplify $\sqrt[4]{\frac{6}{a^2} - 2a/a}$
13. Simplify and express without radicals
   $\frac{\sqrt{x}}{\sqrt[11]{x}}$
14. Simplify $\sqrt{18 - 3\sqrt{8} + \sqrt{50}}$

Topic III - Linear Equations, Inequalities, and Absolute Value

1. Solve for $x$:
   $x = \frac{m}{n}x + 2$
2. Solve for $a$:
   $\sqrt{a - 1} - 2 = 2$
3. Solve for $x$:
   $\frac{2 - \frac{2}{x}}{2x + 1} = \frac{1}{x + 1}$
4. Solve the system of equations for $x$ and $y$:
   $3x + 5y = -7$
   $4x - 2y = 8$
5. Solve the system of equations for $x$ and $y$:
   $5x = 3y$
   $3x + 2y = 21$
6. If $a = -2$ then evaluate:
   $|a - 1| - |a| + 1$
7. Find the equation of the line in the figure shown to the right.
8. Solve for $x$:
   $|4 - 3x| \leq 7$
**Topic IV - Polynomials and Polynomial Equations**

1. Solve for $x$:
   \[ 3x + 5x^{1/2} - 28 = 0 \]

2. Solve for $x$:
   \[ -2x^2 + 2x + 1 = 0 \]

3. Solve for $x$:
   \[ (x-1)(x-2) = 1 \]

4. Solve for $x$ by completing the square
   \[ 4x^2 + 3x - 1 = 0 \]

5. Solve for $x$:
   \[ \sqrt{2x - 5} - \sqrt{x - 2} = 2 \]

6. Divide $x^3 - 10x + 3$ by $x + 3$

7. Verify that 2 is a root of the polynomial
   \[ 6x^3 - 19x^2 + 9x + 10 \]

8. Find all values of $a$ so that the polynomial
   \[ ax^2 + 5x + 2 \]

and factor this polynomial completely.

9. Solve for $x$:
   \[ x^2 + 2x - 3 > 0 \]

10. Graph the equation:

**Topic V - Functions**

1. \( f(u) = \frac{-3u^2 + 2u + 3}{au^2 + bu + 1} \). Find $f(0)$.

2. \( f(x) = -\frac{3}{x - 4} \). Find $f(\frac{1}{x + 2})$ and simplify.

3. \( f(x) = 3x + 1, \ g(x) = x^2 - 1 \).

4. If \( f(x) = \frac{2x - 1}{x - 1} \), for which $x$ does $f(x) = -5$?

5. \( f(t) = \frac{2t - 1}{t + 3} \). Find $\frac{5}{f(3)}$.

6. Graph the function \( f(x) = \frac{1}{x - 3} \).

7. Graph the function \( h(x) = |x^2 - 1| \).

8. Find the domain and range of the function
   \[ g(x) = \sqrt{10 + 2x - x^2} \]
Topic VII - TRIGONOMETRY

1. Given \( \theta \) as shown in the figure to the right, find \( \sin \theta \).

2. Given \( \theta \) as shown in the figure to the right, find \( \tan \theta \cos \theta \).

3. Verify the identity:
   \[ \tan(\pi - x) = -\tan x \]

4. Verify the identity:
   \[ \sec y = \cos y = \tan y \sin y \]

5. Verify the identity:
   \[ \tan x + \tan y = \frac{\sin(x + y)}{\cos x \cos y} \]

6. Find all \( x \) satisfying:
   \[ \tan^2 2x = 3, \ 0 \leq x \leq \pi \]

7. Graph the equation:
   \[ y = \cos 3x \]

8. Graph the equation:
   \[ y = 3 \sin \frac{x}{2} \]

9. In the figure shown to the right find \( \sec \theta \).

10. If \( 0 < \theta < 2\pi \) find all values of \( \theta \) for which \( 2 \sin \theta = \tan \theta \)

11. Express \( 108^\circ \) in radians

12. \( \cos \frac{\pi}{6} = \)

13. \( \frac{5\pi}{6} = \)
Topic VII - LOGARITHMIC AND EXPONENTIAL FUNCTIONS

1. Solve for $x$: $\log_a x = b$  
2. Solve for $u$: $3^u = 4$

3. Evaluate $\log_4 \sqrt{7}$  
4. Evaluate $\log_4 \left(\frac{y^9}{32}\right)$

5. Which of the following is larger?  
   $5 - \log_2 60$, $3 - \log_2 20$

6. Simplify the following:  
   $\log x^2y - \frac{1}{2} \log x + 3 \log y$

7. Graph the equation:  
   $y = \left(\frac{1}{2}\right)^x$

8. Graph the equation:  
   $y = \left|\log_{10} x\right|$

9. Solve for $x$:  
   $5^{3x} - 1 = 0$

10. Solve for $x$:  
    $\log_{10} \left(3x + 1\right) = 3$

Topic VIII - WORD PROBLEMS

1. Sue is 2 years older than John. 15 years ago she was twice as old as he was. If $x$ and $y$ are the ages of Sue and John now, give a system of equations that could be solved to find $x$ and $y$.

2. A 3x5 photograph is enlarged so that its width measures 7". What is the length of the enlargement?

3. If the circumference of a circle is multiplied by 5, how much is the area increased?

4. Two numbers add to 17, and 7 times the first minus five times the second is 3. What are the numbers?

5. A positive number is taken to the 1/3 power and the result is squared. The final answer is 9. What is the original number?

6. The sine of twice an angle is $\frac{\sqrt{2}}{2}$. If the angle is between 0 and $2\pi$ what are its possible values?

7. The price of a plane ticket has been increased by 15% to $172.50. What was the cost before the increase?

8. The radius of a circle is increased by 20%. What is the percent increase in area?
Solutions for Sample Test

1. $x + 1$

2. $\frac{-1}{x^2 + 2x}$

3. $\frac{-60u}{v}$

4. $\frac{x + 3}{x(x - 2)}$

5. $\frac{3a - 2b}{(a - b)(a + 2b)(a - 3b)}$

6. $\frac{xy}{x^2 - y^2}$

7. $a^2b$

8. $\frac{x^2}{x - 3}$

Topic II - Exponents and Radicals

1. $\frac{1}{8}$

2. $5\sqrt{6}$

3. $(u + v)\sqrt{u - v}$

4. $2\sqrt{4x^2 + 9y^2}$

5. $20a^4b^2$

6. $\frac{\sqrt{x}}{x}$

7. $x^{u^2 - u}$

8. $x^{a^2 - 1}$

9. $a^{\frac{3\sqrt{2}}{2}}$

10. $\frac{x^2y^{7/2}}{9^3}$

11. $\frac{\sqrt{12}}{9x^8}$

12. $-a\sqrt{a}$

13. $x^{9/22}$

14. $2\sqrt{2}$

Topic III - Linear Equations, Inequalities and Absolute Value

1. $\frac{2n}{n - m}$

2. $a = 17$

3. $x = \frac{-2}{3}$

4. $x = 1, y = -2$

5. $x = \frac{63}{19}, y = \frac{105}{19}$

6. $2$

7. $\frac{1}{2}y - \frac{1}{3}x = 1$

8. $-1 \leq x \leq \frac{11}{3}$

9. $a = 17$
Topic IV - POLYNOMIALS AND POLYNOMIAL EQUATIONS

1. \[ x = \frac{49}{9} \]
2. \[ x = \frac{1 \pm \sqrt{3}}{2} \]
3. \[ x = \frac{3 \pm \sqrt{5}}{2} \]
4. \[ x = -1, \ x = \frac{1}{4} \]
5. \[ x = 27 \]
6. \[ x^2 - 3x - 1 + \frac{6}{x + 3} \]
7. \[ (x - 2)(2x + 1)(3x - 5) \]
8. \[ a < \frac{25}{8} \]
9. \[ x < -3 \text{ or } x > 1 \]
10. [Graph]

Topic V - FUNCTIONS

1. \[ f(0) = 3 \]
2. \[ \frac{3x + 6}{4x + 7} \]
3. \[ f(g(x)) = 3x^2 - 2 \]
4. \[ x = \frac{4}{7} \]
5. \[ g(f(x)) = 9x^2 + 6x \]
6. [Graph]
7. [Graph]

8. Domain: \[ 1 - \sqrt{11} \leq x \leq 1 + \sqrt{11} \]
Range: \[ 0 \leq y \leq \sqrt{11} \]
Topic VI - TRIGONOMETRY

1. \( \sin \theta = \frac{2\sqrt{u^2 - 4}}{u^2} \)

2. \( \frac{s}{\sqrt{r^2 + s^2}} \)

3. \( \tan(\pi - x) = \frac{\sin(\pi - x)}{\cos(\pi - x)} = \frac{\sin x}{-\cos x} = -\tan x \)

4. \( \sec y - \cos y = \frac{1}{\cos y} - \cos y = \frac{1 - \cos^2 y}{\cos y} = \frac{\sin^2 y}{\cos y} = \tan y \sin y \)

5. \( \tan x + \tan y = \frac{\sin x}{\cos x} + \frac{\sin y}{\cos y} = \frac{\sin x \cos y + \sin y \cos x}{\cos x \cos y} = \frac{\sin(x + y)}{\cos x \cos y} \)

6. \( x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6} \)

7.

8.

9. \( \sec \theta = -\frac{\sqrt{13}}{3} \)

10. \( \theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3} \)

11. \( \frac{27\pi}{45} \)

12. \( -\frac{\sqrt{3}}{2} \)
Topic VII - LOGARITHMIC AND EXPONENTIAL FUNCTIONS

1. \( x = a^b \)

2. \( u = \frac{\log 4}{\log 3} = 1.26 \)

3. \( x = \frac{1}{4} \)

4. \( x = -\frac{23}{2} \)

5. \( 5 - \log_2 60 \)

6. \( \log(x^{3/2} y^4) \)

7. \( \)

8. \( \)

9. \( x = 0 \)

10. \( x = 333 \)

Topic VIII - WORD PROBLEMS

1. \( x = y + 2 \)
   \( x - 15 = 2(y - 15) \)

2. \( \ell = 11 \ 2/3 \)

3. 25 times

4. 7 1/3, 9 2/3

5. 27

5. \( \theta = \pi/8, \ 3\pi/8, \ 9\pi/8, \ 11\pi/8 \)

6. \( 44\% \)