Theorem 12.20 (A repeated linear factor) *If $P(s)$ and $Q(s)$ are polynomials of degree μ and v, respectively, and $\mu < v + n$ and $Q(a) \neq 0$, then Equation (12-36) becomes*

$$Y(s) = \frac{P(s)}{(s-a)^n Q(s)} = \sum_{k=1}^{n} \frac{A_k}{(s-a)^k} + R(s), \tag{12-38}$$

where R is the sum of all partial fractions that do not involve factors of the form $(s-a)^j$. Furthermore, the coefficients A_k can be computed with the formula

$$A_k = \frac{1}{(n-k)!} \lim_{s \to a} \frac{d^{n-k}}{ds^{n-k}} \frac{P(s)}{Q(s)}, \qquad \text{for } k = 1, 2, ..., n. \tag{12-39}$$

Proof We employ the method of residues. First, multiplying both sides of Equation (12-38) by $(s-a)^n$ gives

$$\frac{P(s)}{Q(s)} = \sum_{j=1}^{n} A_j (s-a)^{n-j} + R(s)(s-a)^n.$$

We can differentiate both sides of this equation $n-k$ times to obtain

$$\frac{d^{n-k}}{ds^{n-k}} \frac{P(s)}{Q(s)} = \sum_{j=1}^{k} A_j \frac{(n-j)!}{(k-j)!} (s-a)^{k-j} + \frac{d^{n-k}}{ds^{n-k}} [R(s)(s-a)^n].$$

In this equation, we take the limit as $s \to a$. We leave as an exercise for you to fill in the steps to obtain

$$\lim_{s \to a} \frac{d^{n-k}}{ds^{n-k}} \frac{P(s)}{Q(s)} = (n-k)!\, A_k,$$

which establishes Equation (12-39).