Theorem 12.21 (Irreducible quadratic factors) *Let P and Q be polynomials with real coefficients such that the degree of P is at most 1 larger than the degree of Q. If T does not have a factor of the form $(s-a)^2+b^2$, then*

$$Y(s)=\frac{P(s)}{Q(s)}=\frac{P(s)}{\left[(s-a)^2+b^2\right]T(s)}=\frac{2A(s-a)-2Bb}{(s-a)^2+b^2}+R(s),$$

where

$$A+iB=\frac{P(a+ib)}{Q'(a+ib)}. \tag{12-40}$$

Proof Since P, Q, and Q' have real coefficients, it follows that

$$P(a-ib)=\overline{P(a+ib)} \quad \text{and} \quad Q'(a-ib)=\overline{Q'(a+ib)}.$$

The polynomial Q has simple zeros at $s=a\pm ib$, which implies that $Q'(a\pm ib)\neq 0$. Therefore, we obtain

$$\operatorname{Res}[Y,a\pm ib]=\lim_{s\to a\pm ib}\frac{s-(a\pm ib)}{Q(s)-Q(a\pm ib)}P(s)=\frac{P(a\pm ib)}{Q'(a\pm ib)}, \tag{12-41}$$

from which we get

$$\operatorname{Res}[Y,a-ib]=\overline{\operatorname{Res}[Y,a-ib]}. \tag{12-42}$$

If we set $A+iB=\operatorname{Res}[Y,a+ib]$ and use Theorem 12.19 and Equations (12-40)–(12-42), then we find that

$$Y(s)=\frac{A+iB}{s-a-ib}+\frac{A-iB}{s-a+ib}+R(s).$$

We then combine the first two terms on the right side of this equation to obtain

$$\frac{(A+iB)(s-a+ib)+(A-iB)(s-a-ib)}{(s-a)^2+b^2}=\frac{2A(s-a)-2Bb}{(s-a)^2+b^2},$$

and the proof of the theorem is complete.