

► **Theorem 12.23 (Heaviside expansion theorem)** Let  $P(s)$  and  $Q(s)$  be polynomials of degree  $m$  and  $n$ , respectively, where  $n > m$ . If  $Q(s)$  has  $n$  distinct simple zeros at the points  $s_1, s_2, \dots, s_n$ , then  $\frac{P(s)}{Q(s)}$  is the Laplace transform of the function  $f(t)$  given by

$$f(t) = \mathcal{L}^{-1} \left( \frac{P(s)}{Q(s)} \right) = \sum_{k=1}^n \frac{P(s_k)}{Q'(s_k)} e^{s_k t}. \quad (12-44)$$

**Proof** If  $P(s)$  and  $Q(s)$  are polynomials and  $s_k$  is a simple zero of  $Q(s)$ , then

$$\text{Res} [F(s) e^{st}, s_k] = \lim_{s \rightarrow s_k} \frac{s - s_k}{Q(s) - Q(s_k)} P(s) e^{st} = \frac{P(s_k)}{Q'(s_k)} e^{s_k t}.$$

This result allows us to write the residues in Equation (12-43) in the more convenient form given in Equation (12-44), and the theorem is proven.