

► **Theorem 12.15 (Shifting the variable s)** If $F(s)$ is the Laplace transform of $f(t)$, then

$$\mathcal{L}(e^{at}f(t)) = F(s - a).$$

Proof Using the integral definition $\mathcal{L}(f(t)) = F(s) = \int_0^{\infty} f(t)e^{-st}dt$, we have

$$\mathcal{L}(e^{at}f(t)) = \int_0^{\infty} e^{at}f(t)e^{-st}dt = \int_0^{\infty} f(t)e^{-(s-a)t}dt = F(s - a).$$