

NAME: KEY

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Show your work on all the problems.

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1. An urn contains 8 white balls and 6 black balls. Balls are selected at random, and without replacement from this urn. What is the probability that the first black ball drawn comes in the fourth draw?

W W W B $E = 4^{\text{th}}$ draw is black

$$n(E) = 8 \times 7 \times 6 \times 6$$

$$n(S) = 14 \times 13 \times 12 \times 11$$

$$P(E) = \frac{8 \times 7 \times 6 \times 6}{14 \times 13 \times 12 \times 11}$$



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2. You are dealt a bridge hand (13 cards). What is the probability that all four suits are included in the hand?

$E =$ All four suits are included
 $E^c =$ at least one of the suits is missing
 Let $A_i =$ the i^{th} suite is missing ($i=1, 2, 3, 4$)

$$P(E^c) = P\left(\bigcup_{i=1}^4 A_i\right) = \sum_{i=1}^4 P(A_i) - \sum_{i < j} P(A_i \cap A_j) + \sum_{i < j < k} P(A_i \cap A_j \cap A_k)$$

$$P(A_i) = \frac{\binom{39}{13}}{\binom{52}{13}} \quad P(A_i \cap A_j) = \frac{\binom{26}{13}}{\binom{52}{13}} \quad P(A_i \cap A_j \cap A_k) = \frac{1}{\binom{52}{3}}$$

$$P(E) = 1 - P(E^c) = 1 - \left\{ 4 \frac{\binom{39}{13}}{\binom{52}{13}} - \binom{4}{2} \frac{\binom{26}{13}}{\binom{52}{13}} + 4 \frac{1}{\binom{52}{3}} \right\}$$

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3. Let A , B , and C be three events. Show that

$$P(A \cap B \cap C) = P(A|B \cap C)P(B|C)P(C).$$

Let $D = B \cap C \Rightarrow P(D) = P(B \cap C) = P(B|C)P(C)$ (*)

Now $P(A \cap B \cap C) = P(A \cap D) = P(A|D) \cdot P(D)$
 $= P(A|B \cap C) \cdot P(B|C) \cdot P(C)$ by (*)