

The second midterm (8 April) will cover the material in the following sections of your text:

3.2 – 3.4, 5.1, 5.2, 6.1

You will have the entire class period to complete the exam. No notes, books or calculators are allowed in the exam. The main topics we have covered are the following:

**SEQUENCES** – Know what a sequence is, what a subsequence is, and how to represent sequences. Know the sum and product notation and how to calculate the kinds of sums and products we have covered.

**RELATIONS** – Know what a relation is, and how to tell if a relation is one of the special kinds of relations we have studied, i.e. reflexive, symmetric, anti-symmetric, and transitive. Know how to tell if a relation is a partial order or an equivalence relation, and the relationship between equivalence relations and partitions.

**DIVISORS & INTEGER REPRESENTATIONS** – Know how to tell if a number is a divisor of another number, the gcd and lcm of two numbers, and how to represent numbers in binary.

**BASIC PRINCIPLES OF COUNTING** – Know the multiplication principle and the addition principle, and how to use these in combination.

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Once you have completed the homework problems you will have a good grasp of the concepts we have covered. Go through the Chapter Reviews and Chapter Self-Tests. Ask yourself if you have any lingering questions about any portion of the material. If you do (and you should!) go back to that portion of the text, read the relevant section and try a few more exercises until you feel confident that you could do them on the exam.

The sample midterm on the next page can be used to gauge how well you have learned the material. Aim to complete the exam in one hour to give yourself some time to check your answers.

If you have any questions come and see me.

Get a good night's sleep the night before the exam, eat well and come to the exam feeling confident that you are well prepared to ace the test!

During the exam proceed deliberately, but *without rushing*. Read each problem carefully so that you know where you are going before you start. You have prepared well and will have plenty of time to finish.

*Sample Exam 2*

1. Let  $a$  be the sequence defined by

$$a_1 = 1, \quad a_{n+1} = a_n + 1, \quad \text{for } n > 1.$$

(a) Find a formula for  $a_n$ .

(b) Find a formula for  $c_k$ , where  $c_k = \sum_{n=1}^k a_n$ .

(c) Find a formula for  $d_k$ , where  $d_k = \prod_{n=1}^k a_n$ .

2. Let  $X$  be the set of positive integers.

(a) Determine whether the relation  $R$  on  $X$  is reflexive, symmetric, anti-symmetric, transitive, and/or a partial order, where

$$(x, y) \in R \quad \text{if} \quad 2 \text{ divides } x + y$$

(b) Let  $p$  be a positive prime number and define the following relation

$$xRy \quad \text{if} \quad x \bmod p = y \bmod p$$

Show that  $R$  is an equivalence relation and describe the equivalence classes.

3. Let  $R$  and  $S$  be relations on  $X$ . Determine whether the following statements are true or false. Justify your answer.

(a) If  $R$  is symmetric, then  $R^{-1}$  is symmetric.

(b) If  $R$  and  $S$  are antisymmetric, then  $R \circ S$  is antisymmetric.

(c) If  $R$  and  $S$  are antisymmetric, then  $R \cap S$  is antisymmetric.

4. Prove that if  $m$  and  $n$  are positive integers, then

$$\gcd(m, n) \cdot \text{lcm}(m, n) = mn$$

5. A collection of books contains 4 history books, 2 math books, and 3 art books.

(a) In how many ways can we select 1 history, 1 math and 1 art book from the collection?

(b) In how many ways can we select two books from different subjects among the collection?

(c) In how many ways can the books be arranged on a shelf?

(d) In how many ways can the books be arranged on a shelf if all the books on the same subject are kept together?